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Shorter Contributions

EMPIRICAL MODELS FOR ATTITUDE VARIABILITY OF THE SPOT 1 SATELLITE

By T. WESTIN
Satellitbild, Swedish Space Corporation

Abstract

Longer sequences of attitude data for SPOT 1 were constructed by concatenation of the attitude information in several consecutive scenes in selected orbits. The attitude time series were used to estimate models for the attitude variogram functions. Explicit expressions for these models are given, which can be used in multi-scene adjustment of SPOT imagery.

INTRODUCTION

THE stabilisation of the SPOT satellite is performed by its attitude and orbit control subsystem (AOCS). Attitude information is supplied by rate integrating gyros, earth sensors and sun sensors. The gyros measure the angular rates about all three attitude axes at 8 Hz frequency, while the earth and sun sensors are used to correct for gyro drift. The attitude information is processed by the onboard computer and is used to control three reaction wheels and two magnetic torquers for keeping attitude angles close to zero. The AOCS was designed to keep the attitude angles less than 0·15° and the attitude angular rates lower than 0·001° s^{-1} (CNES and SPOT Image, 1988).

The attitude angular rate measurements are not only used for stabilisation of the satellite platform, but are also downlinked with the telemetry data stream to the receiving stations and provided on the CCT together with the satellite scene. As only the rates are provided, a constant attitude bias remains unknown, while relative attitude angular displacement can be calculated by integration at 0·125 s intervals. This gives 72 or 73 measurements in a SPOT scene which can be used in the geometric correction of the scene to give a resultant image of very high geometric integrity.

When processing several SPOT scenes in the same satellite pass, the image data can be connected into one single long image, thereby greatly reducing the need for geodetic control in geometric correction. In cases were scenes in the same pass are separated by one or more missing scenes, it is not possible to form this extended image, and the attitude angular displacement between them is unknown. It is still possible to extrapolate or interpolate the geometry over the gap by using orbital constraints and by assuming zero attitude change over the gap (Westin, 1991). The accuracy of extrapolation, and the weighting of the zero attitude change constraint in interpolation, is determined by the statistical properties of attitude change over time. The purpose of this paper is to empirically investigate these properties and to provide attitude variability models that can be used in the geometric adjustments of SPOT scenes.

DATA

Time series of attitude data for SPOT 1 were constructed by concatenating the attitude rate information in several consecutive scenes in satellite passes. Ten time series were constructed from selected orbits in the period 1986 to 1988. The time series were of varying lengths, 28 s to 70 s, representing three to nine scenes each.

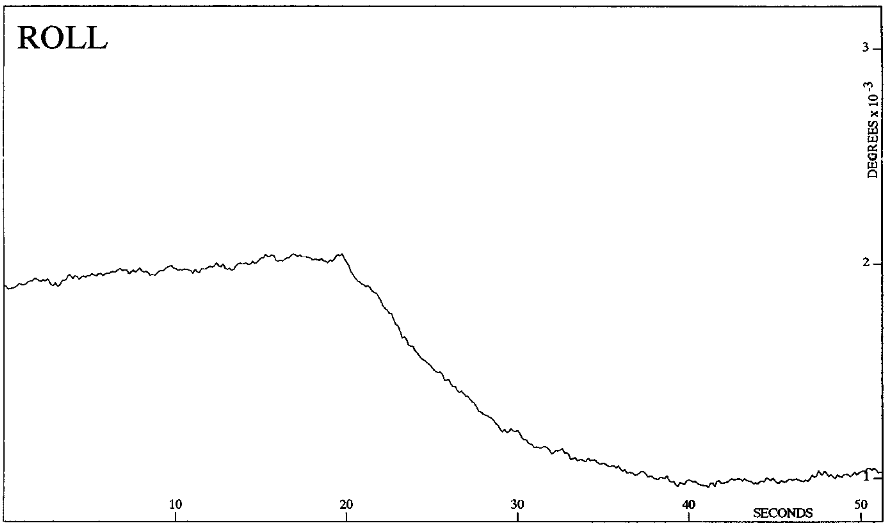

FIG. 1. Example of a roll angle time series.

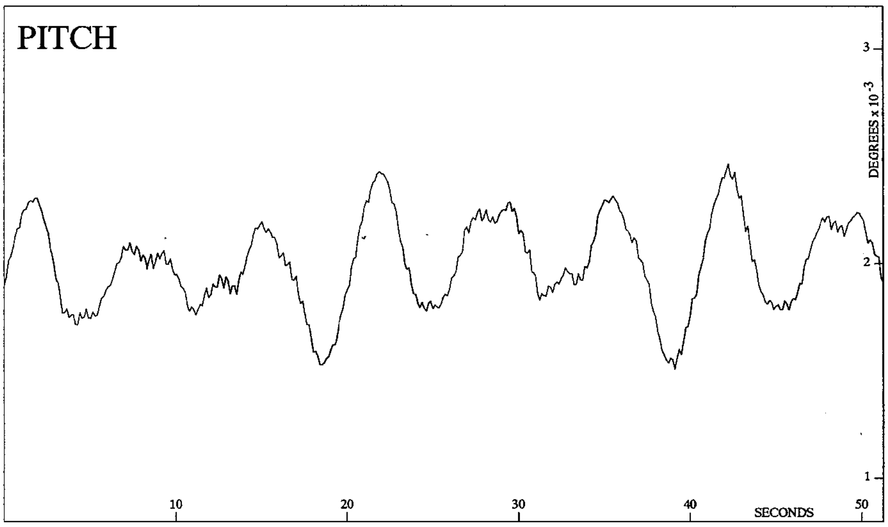

FIG. 2. Example of a pitch angle time series.

Attitude angles were computed at 0·125 s intervals by numerical integration of the attitude rates. Examples of attitude time series are given in Figs. 1, 2 and 3.

ATTITUDE VARIOGRAMS

Consider the attitude time series to be a random function $\theta(t)$. As the attitude angles are derived from attitude rates, there is always an unknown bias in the attitude time series. It is thus not possible to compute statistics such as mean, variance or covariance. It is, however, possible to compute statistics of attitude increments, $\theta(t)-\theta(t+\Delta t)$. The variance of the increments is called the variogram function:

$$2\gamma(t,\Delta t)=E\{[\theta(t)-\theta(t+\Delta t)]^2\}. \qquad (1)$$

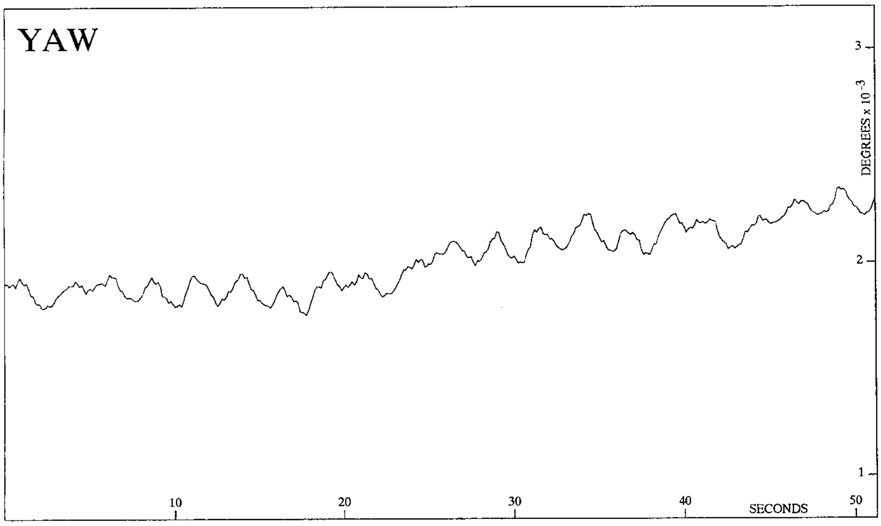

FIG. 3. Example of a yaw angle time series.

Under the hypothesis of second-order stationarity, or under the slightly weaker intrinsic hypothesis (Journel and Huijbregts, 1978), the variogram only depends on the separation Δt and not on the position t. The variogram can then be expressed as:

$$2\gamma(\Delta t)=E\{[\theta(t)-\theta(t+\Delta t)]^2\}. \qquad (2)$$

This is a necessary assumption to make it possible to estimate the attitude variogram, since only one realisation of each time series is available. An estimator, $2\gamma^*$, of the variogram can then be computed as:

$$2\gamma^*(\Delta t)=\frac{1}{N(\Delta t)}\sum_{i=1}^{N(\Delta t)}[\theta(t)-\theta(t+\Delta t)]^2 \qquad (3)$$

where $N(\Delta t)$ is the number of available pairs of attitude data which are separated by the distance Δt.

Equation (3) was used to estimate variograms for roll, pitch and yaw. The results are given in Fig. 4.

VARIOGRAM MODELS

To be useful in the adjustment of SPOT imagery, mathematical models need to be fitted to the estimated variograms. By inspecting the results of Fig. 4, it can be seen that the three variograms are of quite different nature. The roll variogram has a pronounced exponential structure, while in the pitch variogram there is a sinusoidal component that dominates. The yaw variogram also has a weak sinusoidal component, but is primarily linear. These observations lead to the following general variogram model:

$$2\gamma(\Delta t)=A\Delta t^B+C(1-\cos(D\Delta t)) \qquad (4)$$

where A, B, C and D are coefficients to be estimated. Applying this model to the variograms for roll, pitch and yaw (ω, p, κ) in Fig. 4, the following results are obtained:

$$2\gamma_\omega(\Delta t)=1774\Delta t^{1\cdot 8} \qquad (5)$$

$$2\gamma_p(\Delta t)=17500\Delta t^{0\cdot 5}+80000(1-\cos(0\cdot 93\Delta t)) \qquad (6)$$

$$2\gamma_\kappa(\Delta t)=3200\Delta t \qquad (7)$$

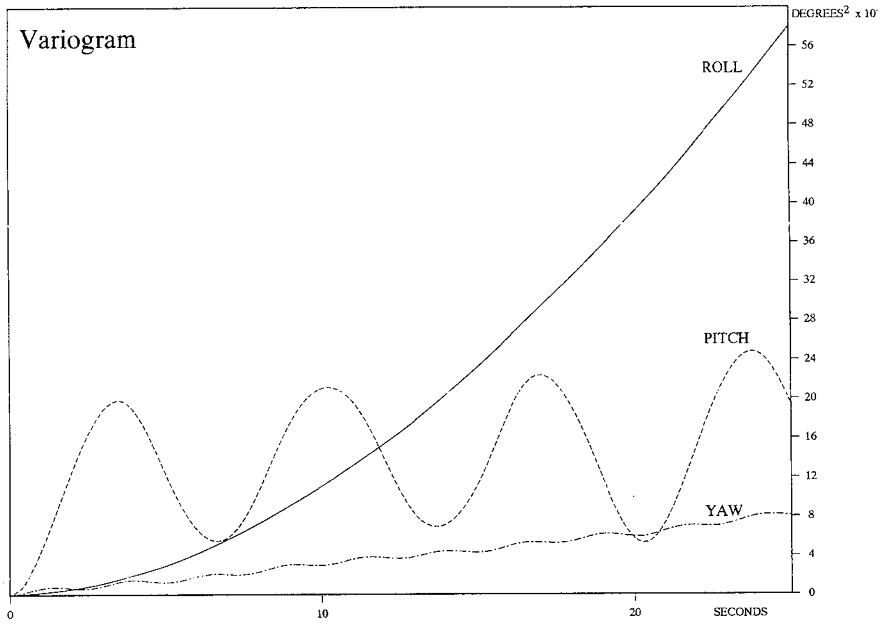

FIG. 4. Experimental variograms for roll, pitch and yaw.

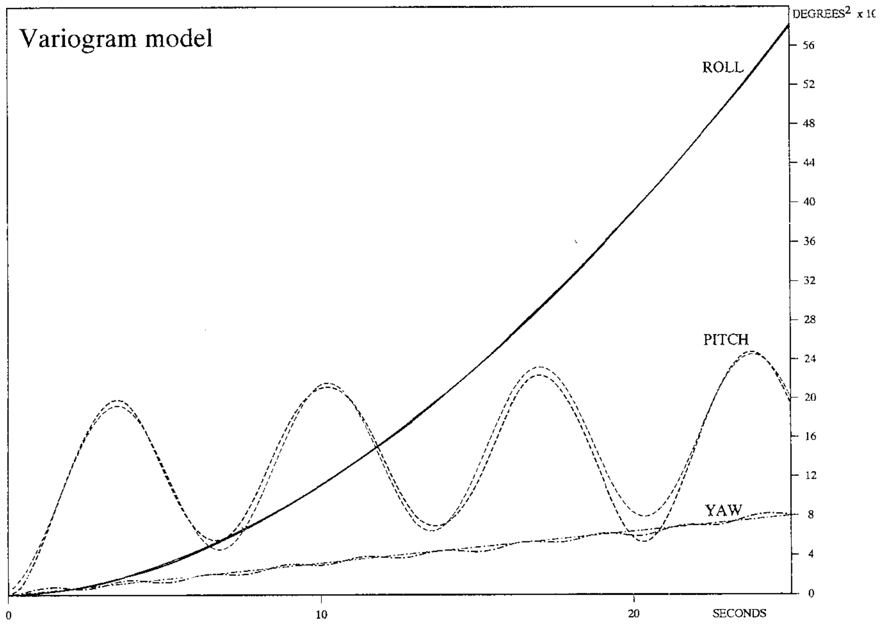

FIG. 5. Variogram models fitted to experimental data.

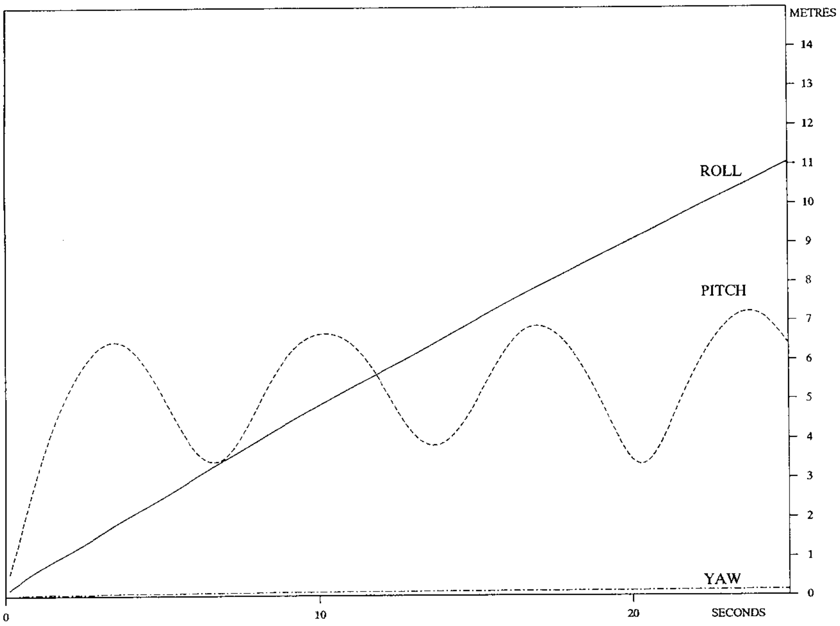

FIG. 6. Magnitude of image displacement caused by attitude angle variations; the 1 σ drift, expressed as metres on the ground, computed from the variograms and plotted as a function of time interval.

where the variograms are expressed in units of $[10^{-6°}]^2$ and Δt in seconds. The variogram models, plotted on top of the computed variograms, are shown in Fig. 5.

DISCUSSION OF RESULTS

The variograms can be used directly for estimating the expected error when bridging over a data gap in pass processing of SPOT scenes. The 1 σ drift in attitude is simply the square root of the variogram for a particular time interval. The roll drift will cause an across track shift of the scene, while the pitch drift causes an along track shift. The yaw drift causes along track shifts which are greatest at the sides of the scene but zero at the centre line. The drift can be transformed to metres on the ground and Fig. 6 shows the 1 σ drift on the ground as a function of time interval. It demonstrates that pitch causes the largest error for gaps up to the size of a scene, while roll drift is the dominant source of error over larger gaps. The yaw drift is insignificant in all practical cases.

As the variograms give the variance of attitude change for a given time interval, their inverse value will directly give the weights for zero attitude change constraints in the adjustment of SPOT scenes separated by a data gap.

REFERENCES

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Résumé

On a pu établir de longues séquences de données d'attitude de SPOT-1 par enchaînement des informations d'attitude relatives à plusieurs scènes consécutives, appartenant à des orbites que l'on a sélectionnées. On a mis au point des modèles dans lesquels les variations d'attitude en fonction du temps apparaissent sous la forme de séries. On explicite les expressions retenues dans cette modélisation; celles-ci peuvent être utilisées pour compenser un ensemble de plusiers scènes SPOT.

Zusammenfassung

Längere Folgen von Orientierungsparametern für SPOT 1 wurden durch Verkettung dieser Informationen in einigen aufeinanderfolgenden Szenen in ausgewählten Bahnen erhalten. Serien des Orientierungs-Zeit-Verhaltens wurden zur Schätzung von Modellen der Orientierungs-Variogramm-Funktionen genutzt. Für diese Modelle wurden explizite Ausdrücke angegeben, die zur Ausgleichung von Multi-Szenen bei SPOT-Aufnahmen genutzt werden können.