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PASS PROCESSING AND EXTRAPOLATION OF SPOT IMAGE GEOMETRY

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Abstract

A mathematical model for pass processing of SPOT imagery is formulated. It is based on the fact that during one pass, the image data stream from each of the instruments in SPOT forms one single very long image. The geometry of this extended image can thus be rectified with as few control points as for only one scene if orbital constraints and attitude measurements are properly taken into consideration. SPOT imagery is, however, not available as this single long image, but only sectioned into scenes. This creates problems which are accounted for in the solution. The possibility of gaps in the sequence of scenes is also taken into consideration. Extrapolation over gaps is of special interest and, by investigating attitude variability, it is shown that extrapolation is successful over multiple scene gaps which is also verified in a real data test.

Introduction

The stability of the SPOT platform and its accurate attitude rate measurements make it possible to rectify a SPOT scene with subpixel accuracy using only one ground control point, while r.m.s. errors of less than 0·5 pixel are achieved by using only a few more points (Westin, 1990). As a SPOT scene covers 3600 km^2, this means that the otherwise proportionally large cost for ground control acquisition can be kept low. While mapping an area covered by more than one scene, further savings are possible. If several scenes are acquired in the same pass, it is possible to take advantage of the fact that the image data stream from each of the instruments in SPOT forms one single very long image. The geometry of this extended image can thus be rectified with almost as few control points as for a single scene.

It would be feasible to apply a mathematical adjustment model for single scenes directly to this extended image. SPOT image data are, however, only available sectioned into scenes. Although digital scenes can easily be merged to form the extended image, it is more convenient to treat the problem as an adjustment of several scenes. The continuous fit of image data is then assured by constraints in the adjustment model. One advantage of this approach is that gaps in scenes are easily handled. If there is a gap of one or more scenes in the pass, a continuous extended image can not be formed, mainly because of missing attitude information. A multi-scene model can handle this simply by changing the constraints.

One important goal is to achieve good accuracy with as few control points as possible, so it is very important to reduce the number of parameters carried in the model to a requisite minimum, thereby avoiding unnecessary degrees of freedom. Great care has therefore been taken to use only those parameters which are justified by the design and specifications of the SPOT mission.

Adjustment Model

A multi-scene adjustment model for scenes in the same pass has been developed as an extension of the single scene adjustment described in Westin (1990). For the latter adjustment it was assumed that, during the time span of one scene, the orbit can with sufficient accuracy be approximated by a plane, circular orbit. The orbital radius is allowed to vary with time, its shape being determined from the satellite ephemeris. In this way, the set of orbital parameters needed in the adjustment of one scene can be reduced to four:

- Ω right ascension of the ascending node;
- t_0 time at the ascending node;
- i inclination; and
- r_0 orbital radius at t_0.

Attitude rate measurements, available at 125 ms intervals, make it possible to compute the attitude variation during the time span of the scene. However, the unknown constant offsets in roll, pitch and yaw at the start of the scene have to be included in the parameter list:

- ω_0 roll;
- ρ_0 pitch; and
- κ_0 yaw.

The validity of the simplified four-parameter orbital model for single scene processing was verified by comparing it to a full orbital model including non-central gravitational forces, showing only insignificant deviations. The three-parameter attitude model was generated by showing that the specified accuracy in the attitude rate measurements was good enough not to introduce any significant errors due to attitude variations within the scene.

To be able to extend this theory to a multi-scene solution, it is necessary to regard the four orbital parameters as corrections to the different sets of estimated values for each scene, rather than as the actual orbital parameters. Start values of the simplified orbital model parameters are estimated from the ephemeris of each scene and hence each scene will give rise to a different set of estimated orbital parameters. By using the same corrections to all scenes, the extended image is kept rigid and the orbital parameters for the whole pass are kept to four.

The same argument also applies to the attitude parameters. The start values for attitudes in the first scene are set to zero. For the following scenes, the start values have to be calculated from the preceding scene. This is possible because the overlap between successive scenes allows an uninterrupted sequence of attitude rate measurements to be constructed. With start values calculated in this way, the same correction vector can be applied to all scenes.

This discussion leads to an adjustment model with only seven parameters for the whole pass:

$$\Delta=[\Delta\Omega\ \Delta t_0\ \Delta i\ \Delta r_0\ \Delta\omega_0\ \Delta\rho_0\ \Delta\kappa_0]^{\mathrm{T}}. \quad (1)$$

For reasons that will be explained below, a more complex model has been chosen where the parameters have been separated into one set applying to the whole pass, and another set applying to each separate scene. If the parameter vector for the whole pass is denoted by $\dot{\Delta}$ and parameter vectors for each scene by $\ddot{\Delta}_j$, the result is:

$$\dot{\Delta}=[\Delta\Omega\ \Delta i\ \Delta r_0]^{\mathrm{T}} \quad (2)$$

$$\ddot{\Delta}_j=[\Delta t_0\ \Delta\omega_0\ \Delta\rho_0\ \Delta\kappa_0]_j^{\mathrm{T}} \text{ where } j=1\ldots \mathrm{m} \quad (3)$$

where m is the number of scenes in the pass. The reason for adding parameters for each scene is to make it also possible to obtain a solution in the cases when there is a gap in the sequence of scenes. In those cases it is not possible to form the continuous sequence of attitude rate measurements and thus it is necessary to allow for a different correction to the attitude angles in the scenes after the gap. The reason for including the time parameter in the scene vector is similar. The universal time (UT) of the first scanline in a scene is only available with millisecond accuracy, which introduces a small uncertainty in the relative positioning of successive scenes. A procedure to overcome this problem for overlapping scenes is developed below, but for scenes after a scene gap it is still necessary to allow for a slightly different correction to the time parameter.

In order to continue to achieve a rigid solution, the parameters have to be constrained. This is done by including observations on them in the adjustment. First, an observation vector with observations on the pass parameters is added:

$$\dot{\mathbf{l}}=[0\ 0\ 0]^{\mathrm{T}} \tag{4}$$

with weights computed from the *a priori* known accuracy in the satellite ephemeris. Similarly, for the first scene in the sequence, observations directly on the scene parameters are added:

$$\ddot{\mathbf{l}}_1=[0\ 0\ 0\ 0]^{\mathrm{T}} \tag{5}$$

with weights computed from the specifications of the attitude control system. For all the following scenes in the sequence, observations on the parameters are not added directly, but on the difference between the parameters in the present scene and the previous scene:

$$\begin{aligned}\ddot{\mathbf{l}}_j&=[\Delta t_0\ \Delta\omega_0\ \Delta\rho_0\ \Delta\kappa_0]_j^{\mathrm{T}}-[\Delta t_o\ \Delta\omega_0\ \Delta\rho_0\ \Delta\kappa_0]_{j-1}^{\mathrm{T}}\\&=[0\ 0\ 0\ 0]^{\mathrm{T}}\text{ where } j=2\ldots m.\end{aligned} \tag{6}$$

The choice of weights for this observation vector depends on whether the scenes are overlapping or separated by a scene gap. In the case of overlapping scenes, the weights are set infinitely high. In this way the parameters are forced to be identical to the previous scene. An adjustment of a sequence of scenes without gaps will thus give the identical result to that obtained when using an adjustment model with only the seven parameters included (1).

When the scenes in (6) are separated by a gap, the weights will be finite. For the attitude parameters, the magnitudes of the weights will depend on the gap size, while the time parameter weight will be independent of the gap size. The sizes of the weights and their dependency on gap size are treated in more detail below.

Each control point will give rise to an observation vector. The observation vector for the i^{th} control point in the j^{th} scene will be:

$$\mathbf{l}_{ij}=[y\ t\ \phi\ \lambda\ h]_{jk}^{\mathrm{T}} \tag{7}$$

where

y=co-ordinate for the (fractional) detector imaging the control point;
t=time for the instant of control point imaging;
ϕ=latitude of the control point;
λ=longitude of the control point; and
h=control point elevation above the ellipsoid.

It should be emphasised that measurement of tie points in the overlap between scenes is superfluous, because the overlap consists of identically the same image in both scenes, and the continuity of image data is assured by the constraints already discussed.

Each control point measurement will give rise to a pair of collinearity equations (Westin, 1990). These express the functional relationship between the observations and the model parameters and can be written in the form:

$$\mathrm{F}(\mathbf{l}_{ij}\ \dot{\Delta}\ \ddot{\Delta}_j)=[0\ 0]^{\mathrm{T}}. \tag{8}$$

Linearisation results in:

$$\mathbf{A}_{ij}\mathbf{v}_{ij}+\dot{\mathbf{B}}_{ij}\ \dot{\Delta}+\ddot{\mathbf{B}}_{ij}\ \ddot{\Delta}_j=\mathbf{f}_{ij} \tag{9}$$

where

$$\mathbf{A}_{ij}=\frac{\partial \mathrm{F}_{ij}}{\partial \mathbf{l}_{ij}}\dot{\mathbf{B}}_{ij}=\frac{\partial \mathrm{F}_{ij}}{\partial\dot{\Delta}}\ddot{\mathbf{B}}_{ij}=\frac{\partial \mathrm{F}_{ij}}{\partial\ddot{\Delta}_j}\ .$$

In addition to the equations of type (9) for each control point, equations are also due to observations on the pass parameters:

$$\dot{\mathbf{v}}-\dot{\Delta}=\dot{\mathbf{f}}, \tag{10}$$

due to observations on the first scene parameters:

$$\ddot{\mathbf{v}}_1-\ddot{\Delta}_1=\ddot{\mathbf{f}}_1 \tag{11}$$

and due to the remaining scene parameter observations:

$$\ddot{\mathbf{v}}_j-(\ddot{\Delta}_j-\ddot{\Delta}_{j-1})=\ddot{\mathbf{f}}_j \qquad j=2\ldots m. \tag{12}$$

Equations (9)–(12) when combined have the form:

$$\mathbf{A}\ \mathbf{v}+\mathbf{B}\ \Delta=\mathbf{f}. \tag{13}$$

The normal equations for this system are:

$$(\mathbf{B}^{\mathrm{T}}(\mathbf{AQA}^{\mathrm{T}})^{-1}\ \mathbf{B})\ \Delta=\mathbf{B}^{\mathrm{T}}(\mathbf{AQA}^{\mathrm{T}})^{-1}\ \mathbf{B}\ \mathbf{f} \tag{14}$$

or

$$\mathbf{N}\ \Delta=\mathbf{t} \tag{15}$$

where

$\mathbf{Q}$=*a priori* cofactor matrix of all measurements.

The matrix $\mathbf{N}$ in the normal equations can be partitioned into submatrices, each of which can be formed in a simple summation process. Its structure can be exemplified for a three-scene case:

$$\mathbf{N}=\begin{bmatrix} \dot{\mathbf{N}}+\dot{\mathbf{W}} & \bar{\mathbf{N}}_1 & \bar{\mathbf{N}}_2 & \bar{\mathbf{N}}_3 \\ \bar{\mathbf{N}}_1^{\mathrm{T}} & \ddot{\mathbf{N}}_1+\ddot{\mathbf{W}}_1+\ddot{\mathbf{W}}_2 & -\ddot{\mathbf{W}}_2 & 0 \\ \bar{\mathbf{N}}_2^{\mathrm{T}} & -\ddot{\mathbf{W}}_2 & \ddot{\mathbf{N}}_2+\ddot{\mathbf{W}}_2+\ddot{\mathbf{W}}_3 & -\ddot{\mathbf{W}}_3 \\ \bar{\mathbf{N}}_3^{\mathrm{T}} & 0 & -\ddot{\mathbf{W}}_3 & \ddot{\mathbf{N}}_3+\ddot{\mathbf{W}}_3 \end{bmatrix} \tag{16}$$

where $\dot{\mathbf{W}}$ is the weight matrix for the pass parameter observations, and $\ddot{\mathbf{W}}_j$ are the weight matrices for the scene parameter observations. The remaining matrices are formed by summation on control points, where the number of control points in scene j is n_j:

$$\dot{\mathbf{N}}=\sum_{j=1}^{m}\sum_{i=1}^{n_i}\dot{\mathbf{B}}_{ij}^{\mathrm{T}}\ \mathbf{W}_{ij}\dot{\mathbf{B}}_{ij} \tag{17}$$

$$\ddot{\mathbf{N}}_j=\sum_{i=1}^{n_i}\ddot{\mathbf{B}}_{ij}^{\mathrm{T}}\ \mathbf{W}_{ij}\ddot{\mathbf{B}}_{ij} \tag{18}$$

$$\bar{\mathbf{N}}_j=\sum_{i=1}^{n_i}\dot{\mathbf{B}}_{ij}^{\mathrm{T}}\ \mathbf{W}_{ij}\ddot{\mathbf{B}}_{ij} \tag{19}$$

where

$$\mathbf{W}_{ij}=(\mathbf{A}_{ij}\mathbf{Q}_{ij}\mathbf{A}_{ij}^{\mathrm{T}})^{-1} \tag{20}$$

and where $\mathbf{Q}_{ij}$ is the *a priori* cofactor matrix for the observation of the i^{th} control point in the j^{th} scene. The vector $\mathbf{t}$ in the normal equations can be partitioned similarly:

$$\mathbf{t}=\begin{bmatrix} \dot{\mathbf{t}}-\dot{\mathbf{W}}\dot{\mathbf{f}} \\ \ddot{\mathbf{t}}_1-\ddot{\mathbf{W}}_1\ddot{\mathbf{f}}_1+\ddot{\mathbf{W}}_2\ddot{\mathbf{f}}_2 \\ \ddot{\mathbf{t}}_2-\ddot{\mathbf{W}}_2\ddot{\mathbf{f}}_2+\ddot{\mathbf{W}}_3\ddot{\mathbf{f}}_3 \\ \ddot{\mathbf{t}}_3-\ddot{\mathbf{W}}_3\ddot{\mathbf{f}}_3 \end{bmatrix} \tag{21}$$

where

$$\dot{\mathbf{t}}=\sum_{j=1}^{m}\sum_{i=1}^{n_i}\dot{\mathbf{B}}_{ij}^{\mathrm{T}}\ \mathbf{W}_{ij}\mathbf{f}_{ij} \tag{22}$$

and

$$\ddot{\mathbf{t}}_j=\sum_{i=1}^{n_i}\ddot{\mathbf{B}}_{ij}^{\mathrm{T}}\ \mathbf{W}_{ij}\mathbf{f}_{ij}. \tag{23}$$

Solving the normal equations for Δ is straightforward. The matrix $\mathbf{N}$ in (16) has a banded-bordered structure which could be taken advantage of, but the size of the system will, in nearly all cases, be small enough to make a direct solution feasible.

Sub-millisecond Positioning of Scenes

As mentioned above, the UT for the first scanline in a scene is only available to the nearest millisecond. For overlapping scenes this is no problem, because the time can be transferred from the previous scene by identifying identical scans in the overlap. This is done most easily by comparing the attitude measurements in the two scenes in the overlap. The relative time between overlapping scenes can thus be exactly determined and the weight for the observation on Δt_0 in (6) is set to infinity.

In the case where the scene is separated from the previous scene by a gap, it is not possible to transfer the time. The $\pm 0{\cdot}5$ ms uncertainty in the UT corresponds to an r.m.s. error of 0·3 ms. As the observation on Δt_0 in (6) is the difference between scene j and $j-1$, both having the same uncertainty, the r.m.s. error of the observation will be 0·4 ms. This corresponds to 2·7 m on the ground.

Weights for Attitude Difference Observations

The observations on attitude differences in (6) have to be weighted. If the scenes are overlapping, the weights will all be infinite. When the scenes are separated by a data gap, there is an unknown drift in attitude between the scenes and the weights then have to be related to the expected drift. Empirical variogram models for the attitudes in SPOT 1 are computed in Westin (1991):

$$2\gamma_\omega(\Delta t) = 1774\ \Delta t^{1{\cdot}8} \tag{24}$$

$$2\gamma_\rho(\Delta t) = 17500\ \Delta t^{0{\cdot}5} + 80000(1 - \cos(0{\cdot}93\ \Delta t)) \tag{25}$$

$$2\gamma_\kappa(\Delta \mathrm{t}) = 3200\ \Delta t \tag{26}$$

where the variograms are expressed in units of $[10^{-6\circ}]^2$ and Δt in seconds. As the variogram is the variance of attitude increments as a function of time interval, the weight is given directly by the inverse of the variogram, evaluated for the time interval of the data gap.

Rectification Results

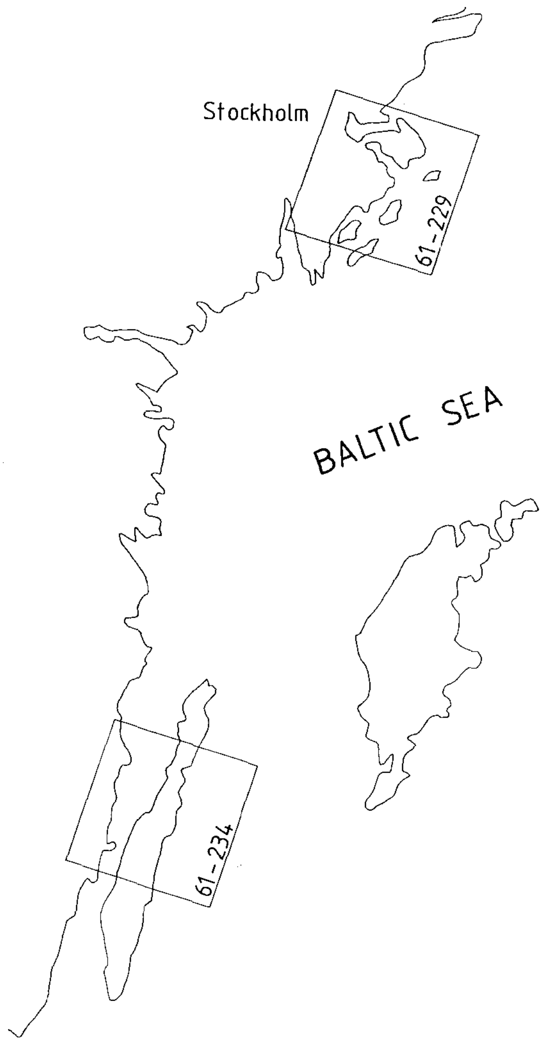
The accuracy of this model for extrapolation was investigated in a test case. Two panchromatic SPOT scenes were used. The first, at node 61-229, and the second, at node 61-234, were separated by a gap of four missing scenes (Fig. 1), corresponding to 210 km on the ground, or 31 s in time.

Two ground control points were collected in the southern scene (61-234) to be used in the adjustments. No ground control was used in the northern scene. The geometry of the scenes after rectification was then evaluated by the use of a large number of checkpoints, 84 in the northern and 43 in the southern. The three dimensional co-ordinates of the checkpoints were considered error free. Given the adjusted model and the three dimensional co-ordinates of the checkpoints, it was possible to compute the expected image co-ordinates of each checkpoint. By identifying the checkpoint in the image, their real image co-ordinates were obtained and hence the error as the difference between real and expected. The resulting errors at the checkpoints are given in Table I.

Table I. Rectification errors at the checkpoints.

Scene	*Direction*	*R.m.s. error (m)*	*Bias (m)*	*Standard deviation (m)*
61-229	x	8·9	−8·2	3·5
	y	6·6	5·6	3·6
61-234	x	4·7	−2·7	3·9
	y	4·0	−0·8	4·0

The larger r.m.s. errors in the northern scene are due to a larger bias in x and y. Under the assumption that the adjustment model is valid, the difference in bias



IG. 1. Location of scenes used in the test. The scenes were acquired during the same satellite pass, and by the same HRV, but were separated by 210 km.

hould primarily be caused by the missing information in the scene gap, that is by ıe missing attitude information and by the uncertainty in relative timing. To verify ıat this assumption is reasonable, the size of the bias can be compared with what 'ould be predicted by the attitude variogram models. If the bias is transformed to n along track and an across track component and the amount of drift in roll and itch between the scenes that would give this bias is calculated, the result is:

$$\text{roll drift} = 0{\cdot}51^\circ \times 10^{-3}$$
$$\text{pitch drift} = 0{\cdot}31^\circ \times 10^{-3} \pm 0{\cdot}46^\circ \times 10^{-3}$$

'here the second term in the pitch drift is due to the uncertainty in relative timing. valuating the variogram models (24) and (25) for roll and pitch over the 31 s time, ives us the expected 1 σ variation:

$$[2\gamma_\omega(31)]^{0{\cdot}5} = 0{\cdot}93^\circ \times 10^{-3}$$
$$[2\gamma_p(31)]^{0{\cdot}5} = 0{\cdot}50^\circ \times 10^{-3}.$$

This corresponds well to the measured results and shows that the larger bias in the northern scene is within what might be expected after extrapolation over a gap of this size.

Conclusions

An adjustment model for pass processing of SPOT scenes has been formulated. By keeping the number of adjustment parameters to a minimum, and by orbital and attitude constraints, the need for ground control points can be kept very low. This is important because ground control acquisition costs are high in unmapped areas. Further savings are possible by allowing gaps in the sequence of scenes in the pass. It has been shown in a test that a scene can be rectified with subpixel accuracy without any control points at all, by extrapolating the geometry from a scene 210 km away. This not only lowers the cost for ground control acquisition and scene purchase, but it also enables the rectification of scenes in inaccessible areas, where control point acquisition is impossible, by extrapolation from scenes in nearby accessible areas.

References

WESTIN, T., 1990. Precision rectification of SPOT imagery. *Photogrammetric Engineering and Remote Sensing*, 56(2): 247–253.

WESTIN, T., 1991. Empirical models for attitude variability of the SPOT 1 satellite. *Photogrammetric Record*, 13(78): 917–922.

Résumé

On a établi un modèle mathématique permettant de traiter l'imagerie SPOT au cours d'un même passage du satellite. Ce modèle est basé sur le fait qu'au cours d'un même passage, l'ensemble des données recueillies par chacun des instruments de SPOT constitue une seule et même image.

On peut ainsi corriger la géométrie de cette longue image à l'aide d'un aussi petit nombre de points d'appui que pour une simple scène, à condition de tenir compte convenablement des contraintes d'orbite et des déterminations d'attitude. Mais les images SPOT ne sont toutefois pas disponibles sous la forme d'aussi longues scènes: elles se trouvent découpées en scènes successives, ce qui crée des problèmes dont on a pu tenir compte dans la résolution. On a également considéré le cas où des lacunes apparaissent dans la séquence des scénes, Il est particulièrement intéressant de pouvoir extrapoler au delà des scènes manquantes et l'auteur montre, en examinant les variations d'attitude, que cette extrapolation réussit, même dans le cas d'une interruption portant sur de nombreuses scènes. Un essai sur des données réelles a d'ailleurs permis de le vérifier.

Zusammenfassung

Für die Verarbeitung von SPOT-Durchgängen wird ein mathematisches Modell formuliert. Es basiert auf der Tatsache, daß während eines Durchgangs der Bilddaten-Strom von jedem der SPOT-Instrumente ein einzelnes sehr langes Bild erzeugt. Die Geometrie dieses ausgedehnten Bildes kann daher mit der gleichen Anzahl von Paßpunkten wie eine einzelne Szene entzerrt werden, wenn die Bahnparameter und die Orientierungsmessungen zweckmäßig einbezogen werden. SPOT-Abbildungen sind jedoch nicht als ein solches langes Bild sondern nur als einzelne Szenen erhältlich. Das schafft Probleme, die bei der vorliegenden Lösung berücksichtigt werden. Das mögliche Auftreten von Lücken wird bei der Szenen-Folge ebenfalls in Erwägung gezogen. Das Extrapolieren über Lücken ist von speziellem Interesse und es wird durch Analyse der Orientierungsänderungen gezeigt, daß die Extrapolation erfolgreich bei Vielfach-Szenen-Lücken angewandt werden kann, was durch reale Testdaten nachgewiesen werden konnte.